INDIAN MARITIME UNIVERSITY
(A Central University, Government of India)

June 2017 End Semester Examinations
B. Tech (Marine Engineering – First Semester)

Mathematics – I – UG11T 2102/UG11T 1102
(AY 2009 – 2014 onwards)

Date: 29.06.2017 | Maximum Marks : 100
Time: 3 Hrs | Pass Marks : 50

Note: i. Use of approved type of scientific calculator is permitted.
ii. The symbols have their usual meanings.

Part-A (10x3=30 Marks)

(All Questions are Compulsory)

Q.1 (a) Prove that the shortest distance between two points in a plane is a straight line.

(b) Find radius of curvature at (1, -1) of the curve $y= x^2 - 3x+1$.

(c) Find the saddle points of the function $x^3 +3xy + y^3$.

(d) Find the n^{th} derivative of $\frac{x}{(x+9)(2x+5)}$.

(e) Show that $\bar{F} = yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k}$ is the solenoidal and irrotational vector field.

(f) Find the length of the arc of the parabola $y^2 = 4ax$ measured from the vertex (0, 0) to one extremity of latus rectum (a, 2a).

(g) Prove that $[\bar{a} \quad \bar{b}+\bar{c} \quad \bar{a}+\bar{b}+\bar{c}] = 0$

(h) Determine the poles of the function $f(z)=\frac{z^2}{(z+1)^2(z-2)}$ and the residue at each pole.

(i) Prove that $\lceil n+1 = n\lceil n$

(j) Evaluate $\int_0^1\int_0^y xy\,e^{-x^2}\,dxdy$

Part-B (5x14=70 Marks)

(Answer any 5 of the following)

Q.2 (a) If $y_n = \frac{d^n}{dx^n}(x^2-1)^n$, show that

$$(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$$

(b) Find the asymptotes to the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$.

(7 + 7=14 Marks)

Q.3 (a) If u = log $(x^3 + y^3 + z^3 - 3xyz)$, show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$

(b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

and $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{-\sin u \cos 2u}{4 \cdot cos^3 u}$

(7 + 7=14 Marks)

Q.4 (a) Investigate for what values of K and W the simultaneous equations

x + y + z = 6; x + 2y + 3z = 10; x + 2y + Kz = W

have (i) no solution,

(ii) a unique solution ,

(iii) an infinite number of solutions.

(b) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 9 & -8 \\ 5 & -4 \end{bmatrix}$

(8 + 6= 14 Marks)

Q.5 a) Find the length of arc of the curve $x = ae^t \text{Sint}, \; y = ae^t \text{Cost}$ from t=0 to t=1.

b) Apply the rule of differentiation under integral sign to prove

$$\int_0^{\pi/2} \frac{\log\left(1+a\sin^2 x\right)}{\sin^2 x} dx = \pi\left(\sqrt{1+a}-1\right)$$

(7 + 7=14 Marks)

Q.6 a) Find the volume of the solid generated by the revolving region bounded by $y=x^3, y=0$ and $x=2$ about the x-axis.

b) Evaluate $\iiint_V dxdydz$ where V is the volume bounded by cylinder $x^2+y^2=4$ and the planes y + z = 4 , z = 0.

(8 + 6=14 Marks)

Q.7 a) Find the directional derivative of $\phi = xy^2 + yz^2$ at $(1,-1,1)$ along the vector $\hat{i}+2\hat{j}+\hat{k}$.

b) If the vector field $\bar{F}=(x+2y+az)\,\hat{i}+(bx-3y-z)\,\hat{j}+(4x+cy+2z)\hat{k}$ is irrotational, find the values of a, b, c to determine the scalar function ϕ such that $\bar{F}=\nabla\phi$.

(7 + 7=14 Marks)

Q.8 a) Evaluate $\oint_C \frac{z+3}{(z-1)^2(z+2)} dz$ where C is the circle $|z|=3$

b) Let $f(z)=u+\hat{i}\,v$ be an analytic function and if u = -3x + 2xy, then find v and express f(z) in terms of z. (7 + 7=14 Marks)

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